PROJECT NOTES/ENGINEERING BRIEFS

APPLICATION OF A REAL-TIME HADAMARD TRANSFORM NETWORK TO SOUND SYNTHESIS*

BERNARD A. HUTCHINS, JR.

Electronotes, Ithaca, N.Y. 14850

A Hadamard transform (HT) analyzes a waveform into Walsh function spectral components in the same manner that a fast Fourier transform (FFT) analyzes a waveform into sine wave components. In contrast to the FFT, a real-time HT network is easy to construct from analog components as the flow graph does not involve complex number arithmetic. The separated Walsh spectral components can be manipulated in a variety of ways before applying them to the inverse transform and time ordering the output for an audio signal.

INTRODUCTION: Some methods of music synthesis with Walsh functions were examined in a previous paper [1]. It was suggested that many processes could be performed with or on the Walsh functions, and that these could be considered instead of the more usual Fourier components if there is some advantage or saving by employing the Walsh functions. While the ear does not hear Walsh functions except in terms of their Fourier components and the repetition rate of the patterns in the Walsh functions, certain processes will be subjectively similar. In many cases in electronic music, it is the general nature of the sound synthesis process rather than the exact details of the process that is important. For example, with subtractive synthesis, the variation of a filter's characteristics in time is more fundamental to the process than the time-independent details such as filter type, input waveform, or cutoff frequency. In additive synthesis, or in processes that involve analysis into a set of harmonics, Walsh functions may prove to be subjectively similar to the same process done with Fourier harmonics.

An example of such a process is the analysis and resynthesis (with some manipulation of the spectral coefficients) of an input waveform. An analog method of doing the fast Fourier transform (FFT) is possible [2], but this would involve a means of handling the complex number arithmetic. The Hadamard transform (HT) has the distinct advantage that only real numbers are involved. Thus an analog HT board can be considered to analyze a section of a waveform. A similar network performing the inverse transform (HT^{-1}) can then resynthesize the section as altered by coefficient manipulation.

WALSH FUNCTIONS AND THE HADAMARD TRANSFORM

Walsh functions form a complete set of orthonormal functions which are all rectangular. The first eight Walsh functions are shown in Fig. 1. Walsh–Fourier analysis can be used to represent any periodic waveform. When the

signal to be represented is discrete in time (for example, a sampled analog waveform), then the Walsh–Fourier coefficients can be determined from the discrete Walsh transform. A fast computer method of doing the discrete Walsh transform is the HT which is derived in a manner similar to the FFT. A flow graph of the HT for an eight-point input is shown in Fig. 2. This HT gives the Walsh–Fourier coefficients corresponding to the first eight Walsh functions in Fig. 1. The coefficients are in a scrambled order.

The HT network is also its own inverse, except the final (1/8) attenuator is not present on the HT^{-1} outputs. The inputs X are just real physical voltages, hence represent real numbers, and all node voltages in the network are real. The multiplying factors are all real and equal to either $+1$ or -1. Thus the nodes in the flow graph can all be realized with a simple operational-amplifier summer or a differential summer, as required. For an HT network with 2^m input points, $m2^m$ operational-amplifiers are required, and each operational-amplifier has associated with it four resistors. The actual circuits that can be used for the various nodes are shown in Fig. 3.

To put things in perspective, for a discrete set of samples, analysis can be by discrete Walsh transform (DWT) or by discrete Fourier transform (DFT). The DWT is more
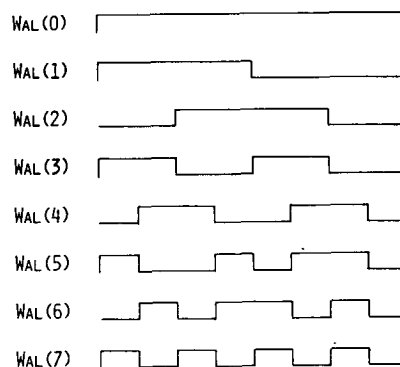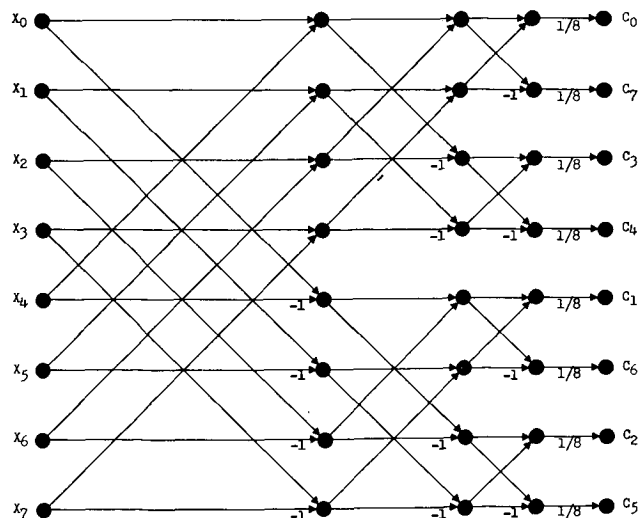

Fig. 1. First eight Walsh functions.

Fig. 2. Flow graph of Hadamard transform.

efficient since it does not use complex numbers, and in cases where the Walsh function set is synchronized with the sampling intervals, the DWT gives an exact representation of the sampled waveform. Both the DWT and the DFT have similar fast computer algorithms, the HT and the FFT, respectively. The saving with these fast algorithms is due to a reduction in the total number of operations.

It is a curious fact that when we choose to analyze a waveform using Walsh functions and an analog method, the HT may not be the most efficient, depending on the number of input points. The DWT is a matrix–vector product, and may be realized with a summing amplifier for each output point. Each output spectral coefficient is a weighted sum ($+1$ or -1) of all the input samples. If the transform consists of 2^m points, the DWT would require only 2^m operational-amplifiers as compared with $m2^m$ for the HT. The tradeoff is seen when the required number of resistors for the two methods is considered. The DWT requires 2^m resistors for summing and two more on each operational-amplifier [3], [4] for each summing point. This means that $(2^m)^2 + 2 \cdot 2^m$ resistors are required. Table I shows the number of components required in both cases. Since the number of resistors required for the DWT goes as $(2^m)^2$, the HT will become more efficient for large networks (probably somewhere between $m = 6$ and $m = 7$). However, for networks of practical size ($m = 3$ or 4), the DWT realization should be used instead of the HT.

To illustrate the DWT realization for eight points, consider the use of the eight-input summer in Fig. 4. The circuit must perform the first-row sum of the matrix–vector product. To find the first Walsh coefficient (corresponding to the Wal (1) content in the waveform), the ordered input samples must be multiplied by either $+1$ or -1 according to the value of Wal(1) in the same segment, and summed.

In a similar manner, the circuit for the C_5 coefficient is given in Fig. 5 as a second example. The other six summers are set up in the same way, except for the Wal(0) coefficient (the dc term) which is an eight-input positive summer.
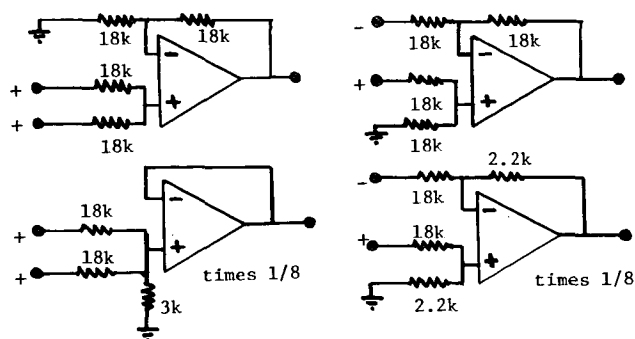
SEQUENCING FOR ANALYSIS IN TIME

Regardless of whether the actual transform is realized by the HT or the DWT, we call it the HT and treat it as a network with eight inputs that represent time samples, and eight outputs that represent the Walsh coefficients.

To be useful for processing audio signals, the inputs must be time ordered. Thus each input is connected to the output of its own sample-and-hold circuit. These are sequenced so that each sample-and-hold takes every eighth sample of the analog waveform that is common to the inputs of all eight sample-and-hold circuits. After the eighth sample is loaded, the correct Walsh analysis of the last eight samples (the last frame) appears at the output of the HT. This analysis can be transferred to additional sample-and-holds so that the analysis of the next frame can start. The necessary sequencing can be provided with a divide-by-eight counter that provides a pulse to trigger the sample-and-hold corresponding to the count. The type 74154 TTL data distributor is convenient for this purpose. The circuit is shown in Fig. 6.

Sample-and-hold circuits can be formed with CMOS analog switches (type 4016) or other convenient circuits. An inexpensive and satisfactory circuit is shown in Fig. 7.

In the actual experimental network, the inverse transform is used as well, and this is also triggered by the same sequencer. The full basic scheme is outlined in Fig. 8.

The operation of the sequencer and connection of the sample-and-hold circuits should be noted. The eight input
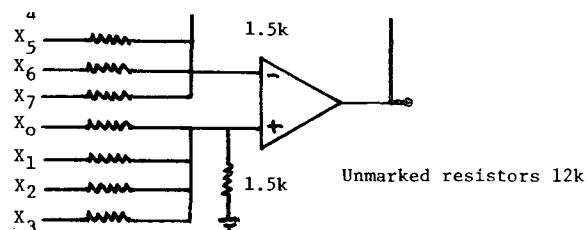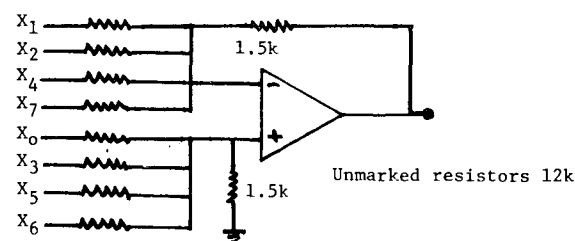
Fig. 4. DWT circuit for C_1.

Fig. 5. DWT circuit for C_5.

Fig. 3. Hadamard transform circuits.

Table I

m	Number of Points	Operational Amplifiers (HT)	Resistors (HT)	Operational Amplifiers (DWT)	Resistors (DWT)
2	4	8	32	4	24
3	8	24	96	8	80
4	16	64	256	16	288
5	32	160	640	32	1088
6	64	384	1536	64	4224
7	128	896	3584	128	16640

sample-and-hold circuits have a common input, and are clocked in sequence as noted above. This loads eight input samples. When the enable pulse on the eighth sample-and-hold falls, this triggers a monostable that provides a single pulse to trigger simultaneously all eight coefficient sample-and-holds. The coefficient sample-and-holds each have one input and one output. Once the coefficients are stored, they are subjected to some form of manipulation. The manipulated coefficients are fed to the HT^{-1} network which synthesizes the first frame of the reconstructed waveform. When the next sequencer pulse arrives, the first input sample-and-hold is loaded with a new value, but this does not change the stored coefficients on the other side of the HT network. At the same time that the first input is reloaded, the first output sample-and-hold is triggered. All output sample-and-hold circuits have a common hold capacitor and a common buffer. The outputs of the HT^{-1} network are thus time ordered by unloading them in sequence as new inputs to the HT network are being loaded. The output voltage of the whole scheme is just the HT^{-1} output last sampled. Observe that if the coefficients are passed directly without manipulation, and circuit errors can be neglected, the output waveform is an exact replica of the input samples delayed by eight clocking intervals.

COEFFICIENT MANIPULATION

In the synthesis of musical tones it is known that the addition of different harmonics, each with its own individual time dependence, is useful. If the coefficients are manipulated with multipliers, this time dependence can be applied with envelopes. As a special case, if all the coefficients are held at one, and each of them is then subjected to enveloping before applying them to the HT^{-1} network, an additive synthesis scheme equivalent to the earlier study [1] results. The additional capability of the new system is that this resynthesis can be done from an analyzed waveform. Manipulation of the coefficients by means of envelopes remains a primary use in musical applications of the HT method.

The coefficients can be switched in or out, and this results in a sequency filter. When about half the coefficient lines are opened up, a sort of sequency formant filter results. The position of the sequency peaks relative to input frequencies can be controlled by varying the clocking rate. In addition, the frequency input to the filter can be varied. Some additional slew rate limiting of the network sample-and-holds can provide additional peaks in the output structure.
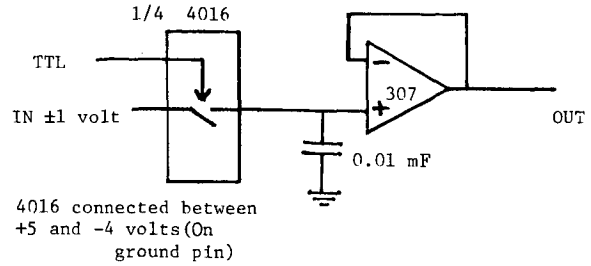

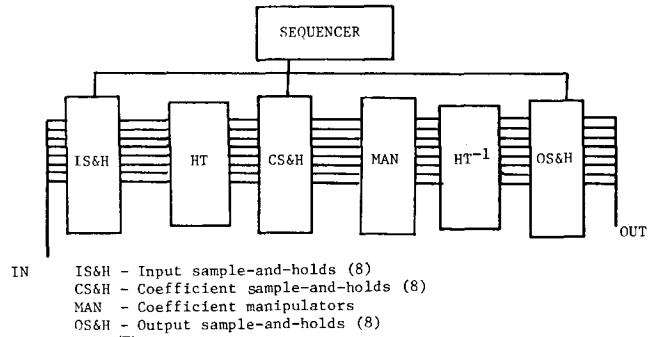Fig. 7. Sample-and-hold circuit.

Fig. 8. Overall Hadamard transform scheme.

RELATIONSHIP BETWEEN THE CLOCK RATE AND INPUT FREQUENCY

The relationship between the clock rate and input frequency is very important. The actual clocking frequency of interest is the frame rate f_f, which is one eighth the clocking rate. In general, small changes in the relationship between the input frequency f_i and f_f can cause great changes in the output.

If f_i and f_f are the same, the coefficients do not change in time, and the result of coefficient manipulation is similar to the case when the coefficients are held at one. When the two frequencies are nearly the same, a variety of "beating" effects are achieved. For example, if the input is a sine wave, a major part of the spectral energy will shift periodically between the Wal(1) and the Wal(2) coefficients as the phase shifts in and out. This opens up the possibility of using these slowly varying coefficients as control signals, since they recover the beat envelope. Integer ratios other than 1:1 do not give constant coefficients, but do give periodic output waveforms if the coefficients are not manipulated in time. Near integer multiples produce complicated beating effects.
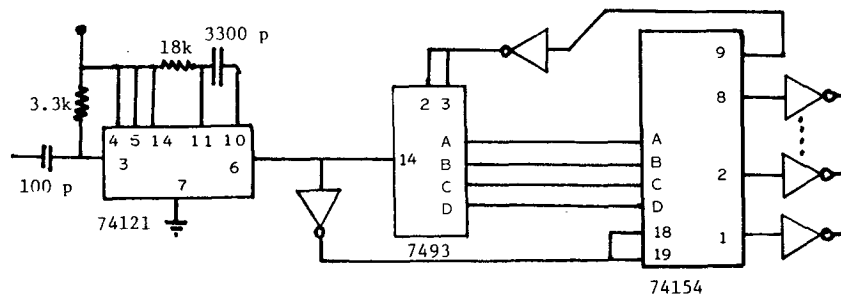

Fig. 6. Sequencer for Hadamard transform.

When f_f is much greater than f_i, the frame is a relatively small portion of the input waveform. Most of the spectral information in this case is carried in the Wal(0) or dc coefficient. If the Wal(0) coefficient is blocked, the frame-to-frame processing can vary greatly and can result in some very complex sounds and changing timbre patterns. On the other hand, when f_f is much smaller than f_i, in general, several cycles of the input will be crowded into one frame, often in violation of the sampling theorem. Output segments may be close to pseudo random in this case. If f_f is in the audio range, however, a pitch corresponding to $8f_f$ will be heard.

ADDITIONAL OUTPUTS

In general, any voltage can be tapped from any point in the network and used as a signal or control voltage as desired. The eventual goal is to obtain musically useful sounds, and nothing should be ruled out. In this regard it should be noted that engineering failures of the network may not be important and may even be useful. A particular example is the violation of the sampling theorem (frequency aliasing) which produces interesting patterns and sequences.

Once analysis is done in terms of Walsh functions, it is possible to use a matrix–vector product to convert to sines and cosines. A very crude sort of Fourier spectrum analyzer is thus possible, but again, it should be considered as a possible control output rather than any sort of scientific measuring instrument. A conversion matrix for up to 16 points is shown in Table II. The accuracy could be improved by using more points in the transform.

In Fourier analysis the power spectrum appears as the Fourier transform of the autocorrelation function. By analogy, if the Walsh coefficients are squared to give the power spectrum, this can be fed to the HT^{-1} network to give what is called the "logical autocorrelation function" [5]. It is easy to convert the logical autocorrelation to the arithmetic autocorrelation function by a simple matrix–vector product. This conversion could be done using operational-amplifier summers on the output of the HT^{-1} networks before the output sample-and-hold. This provides an addi-

tional output that is probably most useful as a control signal, particularly for filters. The conversion scheme is shown in Table III.

FREQUENCY SHIFTING IMPLICATIONS

The possibility that Walsh coefficients can be manipulated to shift the frequency of an input sine wave is implied by the phase dependence of the Walsh coefficients of a sine wave. If the phase angle between a sine wave and Wal(1) is θ, then the C_1 coefficient is calculated as

$$C_1(\theta) = \int_\theta^{\theta + 2\pi} \sin(x - \theta) \, \text{Wal}(1, x) \, dx$$

$$= \int_\theta^{\theta + 2\pi} (\sin x \cos \theta - \cos x \sin \theta) \, \text{Wal}(1, x) \, dx$$

$$= \cos \theta \, C_1(0).$$

In the same manner, $C_2(\theta) = C_1(0) \sin \theta$, $C_5(\theta) = C_5(0) \cos \theta$, $C_6(\theta) = C_5(0)(-\sin \theta)$, etc. Thus if the coefficients are made to vary as either sines or cosines, the phase will be shifted, and this can produce a frequency shift. In the HT network the coefficients would be manipulated by applying the outputs of a quadrature oscillator to the multipliers controlling the coefficients. This frequency shift is illustrated in Fig. 9.

Table III. Conversion from logical to arithmetic autocorrelation.

Arithmetic Autocorrelation	Components of Logical Autocorrelation [4]
R_0	L_0
R_1	L_1
R_2	L_2
R_3	$2L_3 - L_1$
R_4	L_4
R_5	$2L_5 - 2L_3 + L_1$
R_6	$2L_6 - L_2$
R_7	$4L_7 - 2L_5 - L_1$
R_8	L_8
R_9	$2L_9 - 4L_7 + 2L_5 + L_1$
R_{10}	$2L_{10} - 2L_6 + L_2$
R_{11}	$4L_{11} - 2L_9 - 2L_5 + 2L_3 - L_1$
R_{12}	$2L_{12} - L_4$
R_{13}	$4L_{13} - 4L_{11} + 2L_9 - 2L_3 + L_1$
R_{14}	$4L_{14} - 2L_{10} - L_2$
R_{15}	$8L_{15} - 4L_{13} - 2L_9 - L_1$

Table II. Matrix transform between Walsh and sinewave harmonics.

Walsh Harmonic	Sinewave Harmonic							
	First	Second	Third	Fourth	Fifth	Sixth	Seventh	
Wal (1)	0.637	0.0	0.212	0.0	0.126	0.0	0.090	0.0
Wal (3)	0.0	0.637	0.0	0.0	0.0	0.212	0.0	0.0
Wal (5)	−0.264	0.0	0.512	0.0	0.307	0.0	−0.036	0.0
Wal (7)	0.0	0.0	0.0	0.637	0.0	0.0	0.0	0.0
Wal (9)	−0.0525	0.0	−0.342	0.0	0.46	0.0	0.188	0.0
Wal (11)	0.0	−0.264	0.0	0.0	0.0	0.512	0.0	0.0
Wal (13)	−0.127	0.0	0.140	0.0	−0.190	0.0	0.456	0.0
Wal (15)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.637
Wal (17)	−0.0125	0.0	−0.042	0.0	−0.100	0.0	−0.375	0.0
Wal (19)	0.0	−0.0525	0.0	0.0	0.0	−0.342	0.0	0.0
Wal (21)	0.00517	0.0	−0.102	0.0	−0.246	0.0	0.154	0.0
Wal (23)	0.0	0.0	0.0	−0.264	0.0	0.0	0.0	0.0
Wal (25)	−0.260	0.0	−0.154	0.0	0.164	0.0	0.030	0.0
Wal (27)	0.0	−0.127	0.0	0.0	0.0	0.140	0.0	0.0
Wal (29)	−0.627	0.0	0.064	0.0	−0.068	0.0	0.074	0.0
Wal (31)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

RECYCLING THE OUTPUT

If an inverter is placed after the output, and this is fed back to the input, the HT network will oscillate. This can be useful when the coefficients are controlled by multipliers and envelopes, as the oscillation loop is broken when the envelopes are all zero. However, the output is not necessarily controlled by the envelopes in this case, as a loop gain of one must be obtained for any output at all. Thus amplitude of the attack is controlled mainly by the natural buildup of oscillation. On the other hand, decay can be controlled to some extent with the envelopes.

CONCLUDING REMARKS

As of this writing, the HT technique has been shown to be useful for the synthesis of complex sounds and complex sequences of sounds. Several new compositional techniques seem possible with the HT. Further experimentation should lead to additional useful techniques.

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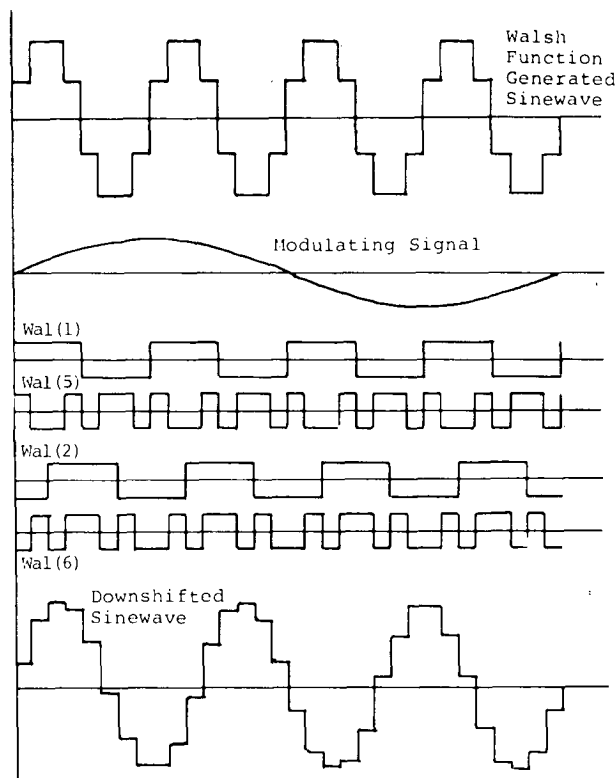

Fig. 9. Frequency shifting by modulation of Walsh coefficients with a quadrature signal.

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SOME USEFUL GRAPHICAL RELATIONSHIPS

RICHARD C. HEYSER

California Institute of Technology, Jet Propulsion Laboratory, Pasadena, Calif.

A recently published mathematical law of energy density partitioning is used to derive some simple graphical relationships that must always be true for a proper physical measurement. In most of the measurements made by engineers, whether of acoustic, electrical, magnetic, or physical properties, it is possible to use these graphical relationships and tell, simply by looking at the data plot; 1) if the data were correctly taken; 2) if the system under analysis is minimum phase; 3) what the other component of a single component plot should be.

INTRODUCTION: Graphs and plots constitute a powerful language which can convey a great deal of information to the knowledgeable engineer. I would like to pass along some graphical relationships which are simple to learn and easy to apply to audio measurements. Armed with such information it is possible to tell a great deal about some system just by glancing at the data plots.

Systems and devices of greatest concern to engineers, whether they are electrical, acoustical, or mechanical, are tested by measuring the cause and effect relationship between an applied stimulus and the observed reaction to that stimulus. If we want to measure everything about the reaction, we must make a detailed observation of its energy density relations. In a previous paper [1] this author presented a principle of energy partitioning which governs such relations in linear passive systems. Stated simply, the positive square roots of the components of energy density partitioning are related by Hilbert transformation. The fact that this principle was presented in a technical paper about loudspeaker reproduction may have obscured for some readers the fact that it is truly a general rule. Some examples of where it may be applied in audio engineering include

1) real and imaginary components of acoustic velocity in damped pipes,

2) resistive and reactive components of acoustic driving-point impedance,

3) In-phase and quadrature components of the frequency response of rooms,

4) amplitude and phase of minimum-phase transfer